

IN THE SPECIFICATION

Please amend the specification as follows:

Please replace page 4 with the following:

fracture simulators. Methods employing PL3D ~~aim at~~ accurately ~~take~~ ~~taking~~ into account geologic layers. One such program ~~presented as PL3D~~, known commercially as GOHFER (GOHFER is believed to be a trademark of Stim-Lab and the Marathon Oil Company), provides grid oriented hydraulic fracture replication capabilities. This grid oriented program, and its mode of operation, is seen in Figure 7. As the front of the fracture moves forward, calculations are made in which each individual grid is either "on" or "off" depending upon whether or not more than half of the individual grid is "covered" by the advancing fracture as it moves outward from the wellbore. If more than one-half of the grid element is covered, then the element is estimated to be fully active. The disadvantage of this system of estimating fracture growth is that it produces too much numerical noise at the fracture tip, and hence in the output data.

Other PL3D methods of simulating fractures include the TerraFrac three dimensional fracturing simulator (TerraFrac is a trademark of the TerraTek Company). This simulator operates as seen in Figure 8, using estimates that are based upon a method of a moving mesh. This method shows less noise than the GOHFER method, because it uses triangle

Please replace page 22 with the following:

fracture width and fracture pressure on each active element. A complete description of the process of the propagation of a hydraulic fracture is thus obtained.

Solutions of the multi-layer equilibrium equations are provided. A three-dimensional body is assumed. The theory also applies to the two-dimensional cases (plane strain, plane stress, antiplane strain). The method provides an efficient way of determining the solution to the equilibrium equations:

$$\sigma_{ij,j} + b_i = 0 \quad (1)$$

for a transversely isotropic elastic medium with a stress strain relationship given by:

$$\sigma_{ij} = C_{ijkl} \epsilon_{kl} \quad (2)$$

In Equation (1) and (2) the index notation is used. In this standard notation, a repeated literal index in any term of an expression implies summation. Therefore, in equation (1)  $\sigma_{ij,j}$  means

$$\sigma_{ij,j} = \sigma_{i1,1} + \sigma_{i2,2} + \sigma_{i3,3} \text{ since it is assumed a 3D body.}$$

The comma preceding an index denotes partial differentiation with respect to that variable represented by that index.;  $u_{i,j}$  thus denotes  $\frac{\partial}{\partial} u_i / \frac{\partial}{\partial} x_j$ .

The notation used in Equation (1) is consequently a shortcut for describing the following equations:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + b_x = 0$$

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + b_y = 0$$

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + b_z = 0$$

Please replace page 25 with the following:

case of isotropic layers, the system (5) has the general solution:

$$\hat{u}_r^l = \sum_j (d_{jr}^l + f_{jr}^l z) e^{\alpha_j^l k z} A_j^l(k) \quad (7)$$

Here  $d_{jr}^l$  and  $f_{jr}^l$  are constants that depend on the material constants of the layer, the  $\alpha_j^l$  are the roots of the characteristic equation for the system of ordinary differential equations, and the  $A_j^l(k)$  are free parameters of the solution that are determined by the forcing terms  $b_i$  in (1) and the interface conditions prescribed at the boundary between each of the layers (e.g. bonded, frictionless, etc.).

Substituting these displacement components into the stress strain law (2), we can obtain the corresponding stress components:  $\hat{\sigma}_{xx}$ ,  $\hat{\sigma}_{yy}$ ,  $\hat{\sigma}_{zz}$ ,  $\hat{\sigma}_{xy}$ ,  $\hat{\sigma}_{xz}$ , and  $\hat{\sigma}_{yz}$ , which can be expressed in the form:

$$\hat{\sigma}_{pq}^l = \sum_j (s_{jpq}^l) e^{\alpha_j^l k z} A_j^l(k) \quad (8)$$

Please replace equation (9) with the following:

$$\hat{\sigma}_{pq}^l = \sum_j (s_{jpq}^l + t_{jpq}^l k_z) e^{\alpha_j^l k_z} A_j^l(k) \quad (9)$$